

# Analytical approach for improving damage equivalence factors



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## ABSTRACT

The fatigue design of bridges under variable amplitude traffic loads is not straightforward. To simplify the task, the current Eurocode provisions apply *fatigue load model 3* and *damage equivalence factors* ( $\lambda$ ). However, the fatigue load model and the damage equivalence factors have several shortcomings: (1) the critical span length is defined for limited bridge influence lines, (2) the damage equivalence factors neglect simultaneous presence of several heavy vehicles on bridges, and (3) the safety margin of damage equivalence factors is not uniform for all bridge influence lines. The current study identifies the effective parameters in the damage equivalence factors through a step-by-step analytical approach. Based on the analytical studies, new propositions for two main parameters of the fatigue load model and *fatigue equivalent length* are made. These modifications not only improve the accuracy of damage equivalence factors (both  $\lambda$  and  $\lambda_{max}$ ) but also extend the application to any bridge influence line. In addition, the proposed modifications are generalized by introducing a partial damage equivalence factor ( $\lambda_s$ ) which takes into account the effect of repetition in influence lines. The proposed parameters are justified with traffic simulations for various bridge cases.

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## 1. Introduction

Fatigue is a major parameter in the design of bridges, and bridge design codes usually have specifications for safety verification against fatigue. The fatigue check of a new bridge is complex since it requires the knowledge of traffic loads during the entire life of the bridge, while load assumptions can be made, the engineers still have the work of doing damage accumulation calculations [1].

The concept of damage equivalence factor for a predefined fatigue load model is proposed by the Eurocodes [2,3] as well as the SIA codes [4,5], to express the traffic actions with equivalent stress range at two million cycles and to compare with the resistance of a detail. It is commonly used due to its simplicity, though very few researches [6,7] have been carried out to evaluate its accuracy. The shortcomings of damage equivalence factors can be summarized as follows [8–10]:

- the definition of *fatigue equivalent length* is limited, and it does not allow for treating all bridge influence lines;
- the effect of simultaneity in which several trucks stand on a bridge simultaneously is neglected;
- the  $\lambda$  factors obtained for different bridge influence lines are widespread;

- the safety margin for some bridge cases are over-conservative and for some cases are non-conservative.

The current study aims to improve the aforementioned shortcomings without changing the concept of damage equivalence factors. To this end, the relationship between the influence lines and the  $\lambda$ -factors is profoundly studied through a step-by-step analytical approach, and the effective parameters are identified. Consequently, some modifications are proposed, and then evaluated by traffic simulations for different bridge types.

## 2. Background of damage equivalence factors

The concept of damage equivalence factors is illustrated in Fig. 1. The left side of Fig. 1 illustrates different elements involved in fatigue verification using damage accumulation. The model of real traffic should be as close to reality, and should comprise the different traffic types for design states. The load history including the dynamic amplification due to the traffic model should be determined for different bridge static systems. The cycles can be extracted from the load history with the Rainflow method. By trial and error, the S–N curve that gives the damage sum equals to 1 can be found. The equivalent force range is the value that corresponds to the two million cycles of the obtained S–N curve.

The right side of Fig. 1 shows the application of the fatigue load model to obtain the force range,  $\Delta F_{FLM}$ , by placing the fatigue load model at the most severe positions. To obtain the same value as the equivalent force range ( $\Delta F_{E2}$ ), which takes into account the

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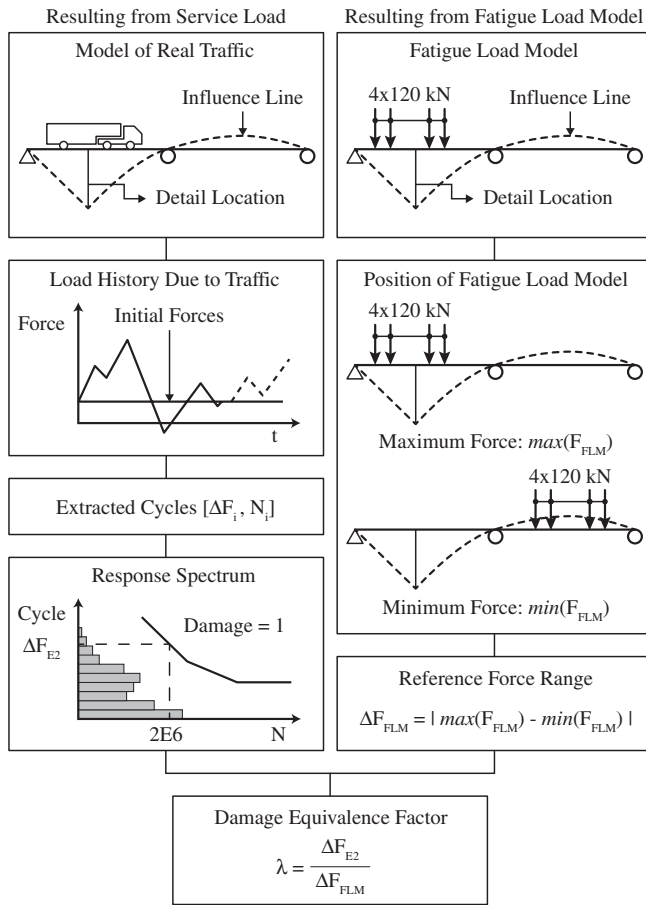


Fig. 1. Concept of damage equivalence factor.

damage accumulation, the value of  $\Delta F_{FLM}$  should be corrected by the damage equivalence factor ( $\lambda$ ). According to the [3],  $\lambda$  can be obtained from:

$$\lambda = \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4, \quad \text{but } \lambda \leq \lambda_{max} \quad (1)$$

where  $\lambda_1$  is a partial factor for the damage effect of traffic (depending on the critical span length of the influence line);  $\lambda_2$  is a partial factor for modification of the traffic volume;  $\lambda_3$  is a partial factor for modification of the bridge design life;  $\lambda_4$  is a partial factor which sums the effect of traffic on the other lanes to the first lane; and  $\lambda_{max}$  is the maximum damage equivalence factor which takes into account the *constant amplitude fatigue limit* (CAFL). The current study only focuses on the single lane damage equivalence factor and maximum damage equivalence factor,  $\lambda_{max}$ .

The fatigue load model 3 (FLM3) of the [2] consists of 4 axles, as shown in Fig. 2, where the weight of each axle is 120 kN. Where relevant, a second set of axles in the same lane should be taken into account. The distance between axles of the second set is similar to the first set, but the weight of each axle is equal to 36 kN (instead of 120 kN). The minimum distance between two vehicles measured from center to center of vehicles is at least 40 m. The basic idea for definition of FLM3 was originally to define a vehicle so

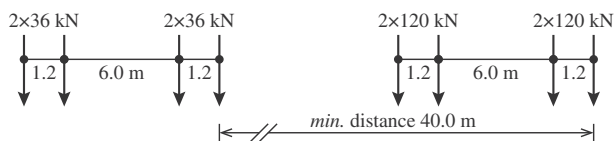


Fig. 2. FLM3 based on the EN 1991-2 [2].

that, assuming  $2 \times 10^6$  times of its passage on a bridge, and after a numerical adaptation with appropriate factors, it gives the same damage as the real traffic during the intended lifetime of the bridge [11].

The SIA codes [4,5] theoretically has the same concept as damage equivalence factors for fatigue verification of bridges. The main differences between the SIA code and Eurocode are in the definition of the fatigue load model and the critical span length (influence length).

Observing the parameters involving in the concept of damage equivalence factors, one can notice that the modifications are limited to two parameters: the fatigue load model, and the fatigue equivalent length.

### 3. Traffic simulation method

A Monte-Carlo traffic simulation program (WinQSIM, [12]) is used to model traffic loads on bridges. The statistical parameters of simulated traffic are based on actual Weigh-In Motion (WIM) measurement from the Götthard station in Switzerland in 2009. To facilitate interpretation of the results, vehicle-by-vehicle traffic simulations are firstly considered for the analytical study. The effect of continuous traffic flow on the damage equivalence factors is important [8,9], and considered in Section 4.5. In addition, for some real bridge influence lines, the continuous traffic simulations are performed in Section 5. For these cases, the hourly heavy vehicle traffic variation is based on the average values measured in different WIM stations in Switzerland in 2009. The traffic is always in *free flow* mode and it is neglected to have traffic congestion on the bridge. The detailed information on traffic simulation parameters and heavy vehicle classes are provided in [13].

Furthermore, the dynamic amplification factor is considered based on the total weight of traffic on the bridge at a given time [14]. When the total weight on the bridge is lower than 300 kN, the dynamic amplification factor is 1.4. When the total weight of traffic on the bridge is more than 1500 kN the dynamic amplification factor is 1.0. For total weights in-between 300 and 1500 kN, this factor is assumed to change linearly. The maximum dynamic amplification factor, 1.4, is chosen conservatively, especially for contemporary bridges with a “good” to “very good” surface quality. For the analytical studies however, for simplification, the dynamic amplification factor is not considered (DAF equal to 1).

The simulations are performed for one year and the number of cycles are multiplied by 100 to extend the cycle response spectra to the intended fatigue design life in the Eurocode. The total number of simulated vehicles is 8'000'000 per year, composed of 25 percent of heavy vehicles (with a minimum gross vehicle weight of 100 kN) and 75 percent of light vehicles.

For simplification, the equivalent force range is calculated by linear damage accumulation with a single slope SN-Curve ( $m = 5$ ) in Section 4; However, for evaluation of real bridge influence lines in Section 5, the fatigue resistance curve of steel is considered, as defined in the [15]: a slope of 3 for cycles with stress ranges higher than constant amplitude fatigue limit (CAFL), and a slope of 5 for cycles lower than CAFL; also, cycles lower than cut-off limit are dismissed.

### 4. Analytical approach

The main goal of analytical approach is to provide some simplifying solutions to improve knowledge about the parameters involved in damage equivalence factors. Through this approach, a meaningful fatigue equivalent length and fatigue load model will be proposed by keeping the simplicity of the damage equivalence

factors. However, this approach does not aim to determine the damage equivalence factors analytically.

The damage equivalence factor is equal to the equivalent force range (moment, shear or reaction) at two million cycles divided by the force range due to the passage of the fatigue load model. The role of the fatigue load model is to calibrate the damage equivalence factors based on the main cycle obtained from the passage of a predefined vehicle on a bridge, thus the first focus in this study is on the equivalent force range.

Both SIA codes and Eurocodes follow a case-by-case definition method that cannot be applied to all bridge types. What theoretically involves the determination of damage equivalence factors is the influence line which results from a bridge type with a certain span length at a detail location. Accordingly, the best definition of fatigue equivalent length is the one directly derived from influence line, which takes into account fatigue characteristics of any influence line such as length, shape, repetitions of shape, difference between maximum and minimum values. The definition of fatigue equivalent length must be simple to apply, since the main objective of the damage equivalence factor method is to simplify the complicated procedure of damage accumulation.

#### 4.1. Imaginary influence lines

In order to simplify the complicated calculations and to focus on different characteristics of influence lines, six imaginary influence lines are chosen as shown in Fig. 3. These imaginary cases do not aim to represent real bridge types, rather each of them is considered for a special purpose that logically could have an effect on the equivalent force range. The I0 case is the base case having a triangular shape. The I1 case, with a trapezoidal shape, is intended to study the shape effect. The I2 case is intended to study the effect of sign change. The I3 case, with five repetitions of the shape I0, is intended to study the repetition effect. The I4 case is intended to study the effect of difference between the minimum or maximum values on the influence line by doubling this range. The I5 case is intended to study discontinuity (sudden changes). The length of each unit (distance between the null points of influence line),  $L_{inf}$ , is equal for all cases and it ranges from 1 m to 100 m. For all cases except I4, the difference between the maximum and minimum values on the influence lines is 1 ( $\Delta_{inf} = 1$ ), where by increasing the unit length it remains constant.

Generally, the equivalent force range (it can be for example moment, shear, reaction, stress, etc.) at two million cycles for a given response spectrum (cycles range and number of cycles) by assuming a single slope SN curve can be calculated as follows:

$$\Delta F_{E2} = \left( \frac{1}{2 \times 10^6} \sum \Delta F_i^m n_i \right)^{1/m} \quad (2)$$

where  $\Delta F_{E2}$  is the equivalent force range,  $\Delta F_i$  is the cycle range  $i$  of the response spectrum,  $n_i$  is the number of the cycles corresponding to  $\Delta F_i$ .

The results of equivalent force range,  $\Delta F_{E2}$  obtained from vehicle-by-vehicle traffic simulations are illustrated in Fig. 4. The results obtained for I0, I2 and I5 are similar in the considered range, indicating the influence lines that have a negative part or sudden sign change can be treated similarly as long as the difference between the maximum and minimum is similar. However, the results for I1, I3, I4 are different from I0.

In the case of I1, having a trapezoidal shape, the equivalent force range increases sharply up to 20 m length and remains constant from 20 m on. This is because the length of vehicles are rarely higher than 20 m, and all axes (regardless of their positions) cause the same effect when  $L_{inf} \geq 20$  m. In addition, the equivalent force ranges obtained for I0 and I1 starting from the same point ( $L_{inf} = 1$  m) converge again by increasing the unit length. The convergence at the two ends are described in Section 4.2.

The I3 case is representative of repetition in the influence line. The equivalent force ranges obtained for I3 at the two limits ( $L_{inf} = 1$  m and 100 m) are higher than the basic case, I0. In the I3 case, for very short as well as very long lengths, the number of cycles are 5 times more than the I0 case, hence the resulted equivalent force ranges will increase by  $5^{(1/5)} = 1.38$  for these points. The mid-range results, however, are highly dependent on vehicles geometry.

In the case of I4, the difference between the minimum and maximum values on the influence line is twice of other cases. Thus, the equivalent force range obtained for I4 is almost twice of the I0 case. Nevertheless, the equivalent force range is studied here, and the effect of difference between the minimum and the maximum values of influence lines can be adapted by a proper fatigue load model, as explained later in Section 4.6.

#### 4.2. Two extreme unit lengths

Since determination of equivalent force range for very short span lengths and very long span is straightforward, the equivalent force range at these extreme lengths are studied first.

For very short lengths ( $L \rightarrow 0$ ) as shown in Fig. 5a, passage of each axle over bridge causes one cycle, providing the influence line has one repetition. For influence lines with more than one repetition, the passage of each axle causes  $N_{inf}$  cycles. Concerning the imaginary cases, passage of each axle over I0, I1, I2, causes one cycle equal to the axle weight ( $\Delta_{inf} = 1$ ). Therefore, the equivalent force range by assuming a single slope SN curve is calculated as follows:

$$\Delta F_{E2, Axle} = \Delta_{inf} \left( \frac{N_{inf}}{2 \times 10^6} \sum q_i^m f_i \right)^{1/m} \quad (3)$$

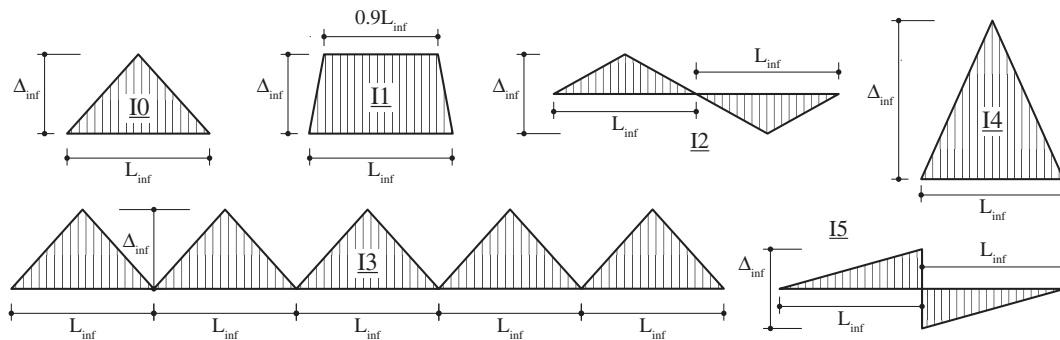


Fig. 3. Imaginary influence lines shape.

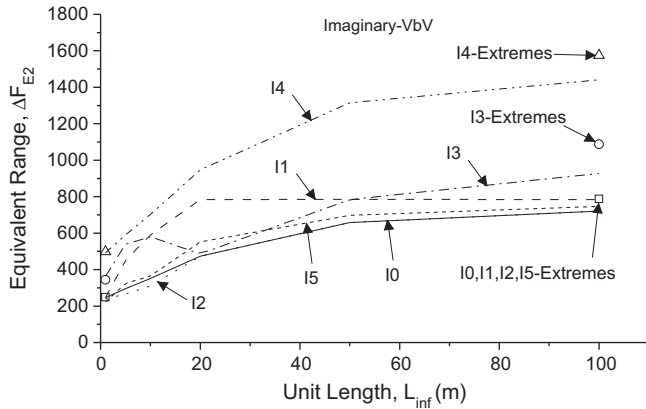
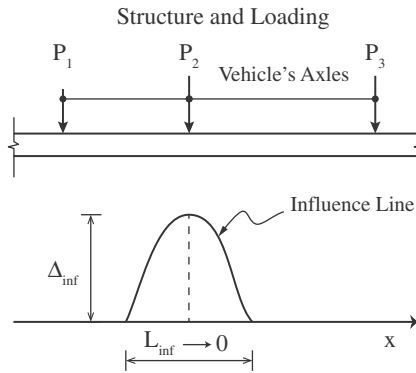
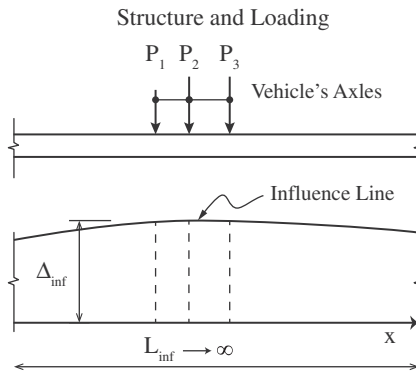


Fig. 4. Equivalent force range for imaginary influence lines and estimations at extreme unit lengths.



(a) Very short span length



(b) Very long span length

Fig. 5. Schematic view of loading and influence lines with extreme unit lengths.

where  $\Delta F_{E2,Axle}$  is the equivalent force range calculated for the axles frequency,  $f_i$  is the frequency (during the whole design life of given bridge) corresponds to the axle intensity  $q_i$ ,  $N_{inf}$  is the total number of repetition in the influence line and  $\Delta_{inf}$  and  $m$  are as already defined. Note that  $N_{inf}$  is a natural number that can be specified by Reservoir or Rainflow cycle counting methods assuming the influence line is the response of a bridge. In the current section for simplification, it is assumed that the cycle ranges obtained from this counting are equal except for Eq. (14) at the end this section which is specifically proposed to treat influence lines with variable cycle ranges with a general approach.

The axle load distribution at the Götthard station in 2009 is illustrated in Fig. 6a. The  $\Delta F_{E2}$  for I0, I1, I2 and I5 can be calculated

by Eq. (3), and it bears 249 kN which is very close to the simulations results for these cases with 1 m length, as shown in Fig. 4. In addition, in the case of I4, the resulting cycle will be twice of the axle weight ( $\Delta_{inf} = 2$ ), thus the equivalent force range will be twice. In the case of I3, the resulted cycles will be equal to the axle weight but, instead of one cycle, five cycles occur ( $N_{inf} = 5$ ). Hence the equivalent force range will be  $5^{1/5}$  time more than I0.

For very long unit lengths ( $L \rightarrow \infty$ ) as shown in Fig. 5b, the small cycles due to passage of each axle can be neglected and we can assume one cycle per each repetition in influence line happens. Regarding the imaginary cases, passage of each vehicle over I0, I1, I2 and I5 causes one cycle equal to the gross vehicle weight ( $\Delta_{inf} = 1$ ), and since the vehicle length is limited and comparing to the unit length,  $L \rightarrow \infty$ , can be assumed like a concentrated load. Similar to  $F_{E2,Axle}$ , the equivalent force range can then be determined as follows:

$$\Delta F_{E2,GVW} = \Delta_{inf} \left( \frac{N_{inf}}{2 \times 10^6} \sum Q_i^m f_i \right)^{1/m} \quad (4)$$

where  $\Delta F_{E2,GVW}$  is the equivalent force range calculated for the GVW frequency,  $f_i$  is the frequency (during the whole design life of given bridge) corresponds to the GVW intensity  $Q_i$ ; also  $N_{inf}$ ,  $\Delta_{inf}$  and  $m$  are as already defined.

The GVW distribution of the Götthard station in 2009 is illustrated in Fig. 6b. The  $\Delta F_{E2}$  for I0, I1, I2 and I5 can be calculated by Eq. (4), and it bears 788 kN which is very close to the corresponding simulations results for I0, I1, I2 and I5 with 100 m length, as shown in Fig. 4. For two other cases, I3 and I4, the values of simulation would tend to the values calculated by Eq. (4), if the unit length went to infinity considering the multiplication factors of  $5^{1/5}$  and 2.0 respectively, as explained for the very short lengths.

It is important to note that vehicle-by-vehicle traffic simulations are considered here. The flat part of equivalent force range for long unit length (in Fig. 4) would not happen if the traffic was continuous in the simulations. In fact, the number of heavy vehicles passing simultaneously on a bridge increases with span length [8,9], which causes an increase in the equivalent force range by enlarging the unit length. In addition, the difference between the maximum and minimum,  $\Delta_{inf}$ , also remains constant by increasing the unit length, which is also another reason why the equivalent force range for very long lengths reaches a fixed value. For many real influence lines (except for reactions)  $\Delta_{inf}$  increases with span length, and as a result, the equivalent force range grows with span length. This last issue can be addressed by a proper fatigue load model.

#### 4.3. Mid-range unit lengths

In the previous section, the equivalent force range is calculated using simplified assumptions that allow us to neglect the vehicle geometry. However, in the case of mid-range lengths, the equivalent force range is influenced by the interaction of parameters including: the vehicle geometry, axle loads, and influence line shape and length. While the vehicle geometries (axles positions and loads) are stochastic, it is not interesting to calculate equivalent force range for every case. An equivalent vehicle which represents all vehicle types (heavy vehicles classes) can be proposed to address this issue. The main assumptions for determining the “equivalent vehicle” are:

1. the total weight of equivalent vehicle equals 1,
2. the small cycles due to the passage of axles are negligible in the total damage sum,
3. for each vehicle, the position of axles are measured from the mass center of the vehicle,

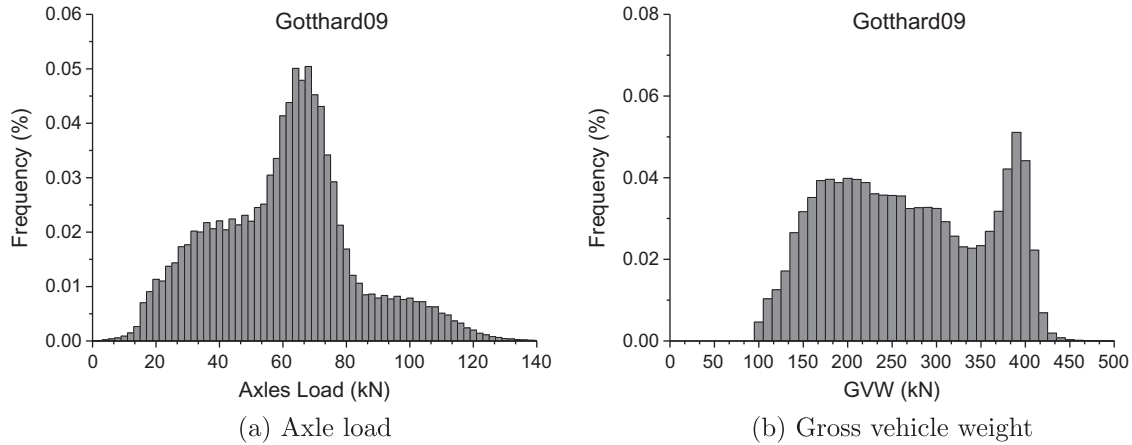


Fig. 6. Frequencies in Götthard WIM station in 2009.

4. the effect of each vehicle in the equivalent vehicle is equal to the ratio of its gross vehicle weight to the total traffic weight,
5. the effect of each vehicle axle load at its corresponding axle position is equal to the ratio of axle load to the gross vehicle weight.

Accordingly, the  $i$ th axles' effect of  $j$ th vehicle in the equivalent vehicle,  $R_{ij}$ , is:

$$R_{ij} = \frac{L_{ij}}{N_{obs} \times Q_{av}} \quad (5)$$

where  $L_{ij}$  is the load intensity of  $i$ th axle, and  $N_{obs}$  is the total number of vehicles (heavy vehicles) and  $Q_{av}$  is the arithmetic mean of gross vehicles weight in the considered traffic. The adjacent axles in certain intervals can be summed to obtain the axle load of the equivalent vehicle within the intervals.

For finding the maximum or minimum forces, generally, the mass center of vehicles should be positioned at the maximum or minimum point of the influence line. In this case, the value of the response due to the set of axles can be calculated as:

$$F_R = \sum_{i=1}^{i=n} L_i [I.L.F_R(x_i)] \quad (6)$$

where  $i$  is the axle index,  $n$  is the total number of axles,  $L_i$  is the  $i$ th axle load intensity and  $[I.L.F_R(x_i)]$  is the influence line ordinates at the abscissa corresponding to the axle position. Note the units of an influence line reflect the units of the response function and the unit load. For example, the units for the influence line of the support reaction and mid-span moment of a simple span bridge are force per force (kN/kN) and moment per force (kN m/kN) respectively.

However, there are some exceptional conditions for which positioning the mass center of vehicles at the maximum or minimum responses will not bear the maximum or minimum responses. For example, when the maximums or minimums are at the start or at the end of influence lines; when the vehicle length (from the first axle to the last axle) is larger than the unit length ( $L_{inf}$ ); or when influence line is discontinuous or its sign changes suddenly. In the current study, the vehicle mass center is always considered as the point for determining the equivalent vehicle, despite its limitations for exceptional cases; nevertheless, the accuracy of such an assumption will be evaluated in the next section.

The equivalent vehicle for the Götthard WIM station in 2009 is determined using the mentioned procedure and is illustrated in Fig. 7, where the adjacent axles are grouped in 1 m intervals. The equivalent force range for the equivalent vehicle is obtained as follows:

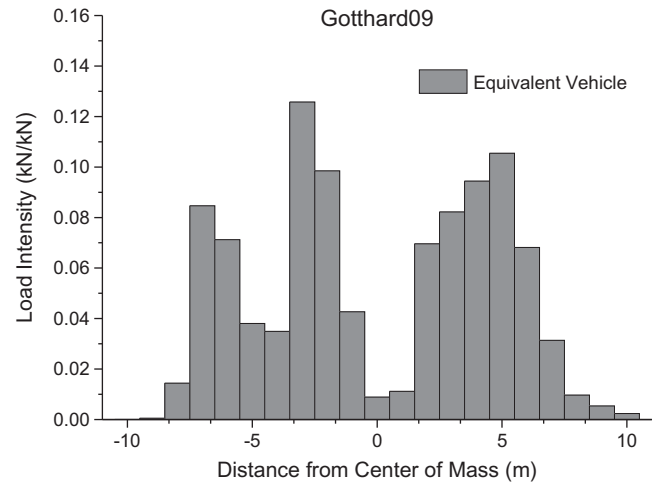


Fig. 7. Equivalent vehicle determined from Götthard WIM station in 2009.

$$\Delta F_{E2,EV} = \Delta F_{R,EV} Q_m \left( \frac{N_{obs} N_{inf}}{2 \times 10^6} \right)^{1/5} \quad (7)$$

where  $\Delta F_{E2,EV}$  is the equivalent force range obtained for the equivalent vehicle,  $\Delta F_{R,EV}$  is the force range due to passage of the equivalent vehicle on the bridge,  $N_{obs}$  is the number of trucks passing over the bridge (during the whole design life),  $Q_m$  is the power mean (computed with exponent  $p = 5$ ) of gross vehicle weight passing on the bridge, and  $N_{inf}$  is number of repetitions in the influence line as already defined. The  $\Delta F_{R,EV}$  parameter has the same unit as the influence line since the weight of equivalent vehicle is 1.0.

The equivalent force range calculated by Eq. (7) is illustrated in Fig. 8 for the imaginary cases; for comparison purpose, the results of vehicle-by-vehicle traffic simulations are also plotted. Fig. 8 shows that the results of equivalent force ranges are properly estimated by Eq. (7) for unit length  $L \geq 20$  m; however, the precision of the equivalent force range resulting from Eq. (7) reduces as the unit length decreases. In fact, the cycles due to passage of axles become more effective as the unit length decreases from 20 m. Accordingly, the equivalent force range tends towards the minimum value as described in Section 4.2.

#### 4.4. Uniformly distributed load with defined length

Although the results obtained from the equivalent vehicle are rather promising, there are still some limitations, including:



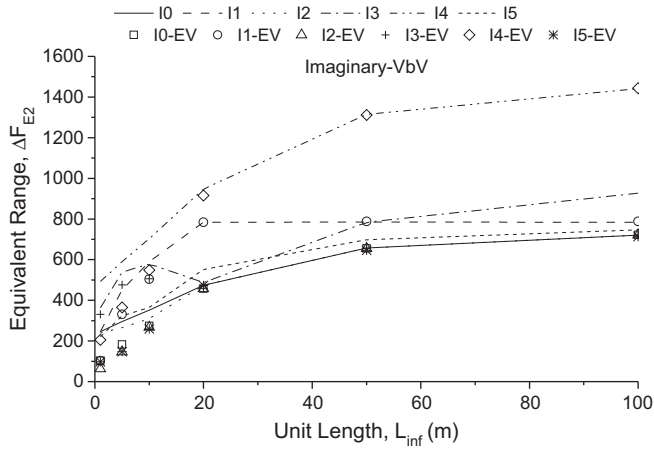


Fig. 8. Equivalent force range determined by equivalent vehicle in comparison with simulation results.

1. the equivalent vehicle is traffic dependent, for instance in the previous section, it is defined for the Götthard traffic in 2009,
2. continuous traffic is neglected in determining the equivalent vehicle,
3. determination of equivalent vehicle is rather difficult for practicing engineers.

An average equivalent vehicle considering several traffic measurements may address the first issue. The same method for determination of the equivalent vehicle is applied for different available WIM measurements within Switzerland in 2009. Then, the average value of different stations within the intervals is calculated. Fig. 9 illustrates the average equivalent vehicle obtained for seven different Swiss traffic WIM in 2009 and the standard deviation of the values within each interval. The load value at each interval partially depends on the traffic station. In order to simplify the equivalent vehicle, the equivalent vehicle is modeled with a uniformly distributed load having a defined length.

The length of the uniformly distributed load can be equal to the average length (axle to axle) of heavy vehicles. The average length of heavy vehicles in the Götthard station in 2009, for example, is about 11.7 m, and the average truck length for all WIM stations in 2009 is about 9.5 m. To evaluate the accuracy of modeling the traffic actions with a distributed load, the uniformly distributed

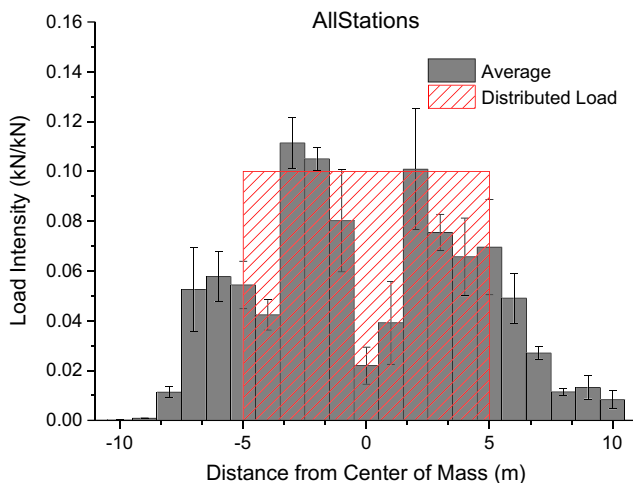


Fig. 9. Average equivalent vehicle and its standard deviation obtained from Swiss traffic.

load with 10 m length and load intensity of 0.1 kN/m/kN (it is divided by its total weight to obtain a unit distributed load) is applied, as shown in Fig. 9. Consider the bridge shown in Fig. 10, which is subjected to a uniformly distributed load of intensity of  $p$ . Also given in Fig. 10 is a segment of an influence line for the response function  $[I.L.F_R]$ . At section  $x$ , an element of the bridge  $dx$  in length is taken, and an element load of  $dP$  can be taken as a concentrated load at point  $x$ . The increment of the response function that results from the load  $dP = p dx$  as:

$$dF_R = p dx [I.L.F_R(x)] \quad (8)$$

where  $[I.L.F_R(x)]$  is the ordinate to the influence line at point  $x$ . Integration of Eq. (8) between A and B gives the total response due to uniformly distributed load  $F_{R,Dist}$ :

$$F_{R,Dist} = p \int_{x_a}^{x_b} [I.L.F_R(x)] dx \quad (9)$$

In the above,  $p$  has been moved outside the integral since it is a constant. Note that  $F_{R,Dist}$  has the same unit as the influence line since  $p$  has unit of kN/m/kN and  $dx$  has unit of m. In the form of Eq. (9), the integral represents the area under the influence line between the limits of points  $x_a$  and  $x_b$ . Thus, the force range due to passage of the distributed load  $\Delta F_{R,Dist}$ , as shown in Fig. 11, is equal to the absolute sum of the area under the influence line by positioning its center at the maximum and minimum values of influence line multiply to  $p$ , providing the unit length,  $L_{inf}$ , be larger than the uniformly distributed load length.

The equivalent force at two million cycles using the uniformly distributed load with defined length,  $\Delta E2,Dist$  can be determined as:

$$\Delta E2,Dist = \Delta F_{R,Dist} Q_m \left( \frac{N_{obs} N_{inf}}{2 \times 10^6} \right)^{1/5} \quad (10)$$

The equivalent force range obtained from Eq. (10) is shown in Fig. 12 for the given imaginary influence lines in comparison with vehicle-by-vehicle equivalent force range resulting from traffic simulation. For the influence lines with unit length,  $L_{inf}$ , lower than uniformly distributed load length (10 m),  $\Delta F_{R,Dist}$  is also determined by passage of uniformly distributed load on the influence line for the sake of having complete range of results at all unit lengths, although this estimation is not correct. Despite many simplifying assumptions, the estimated equivalent force range by the distributed load is acceptable precise, except for the unit length lower than 20 m where the results are mainly influenced by the axles-by-axle passages and the distributed load is not applicable.

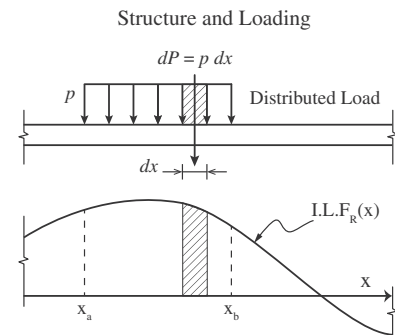


Fig. 10. Parameters for finding maximum response due to uniformly distributed load.

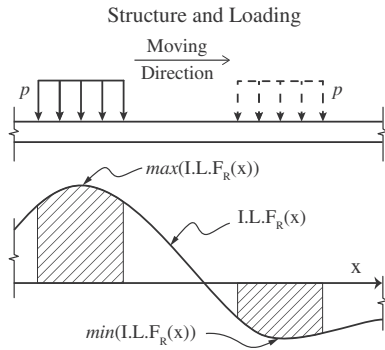


Fig. 11. Parameters for finding force range due to passage of uniformly distributed load with a defined length.

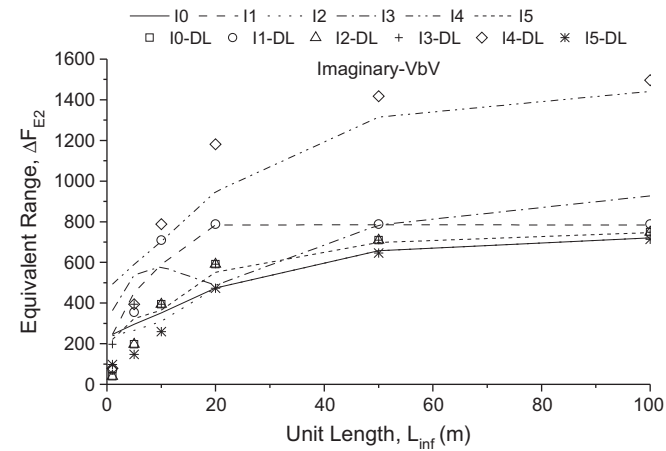


Fig. 12. Equivalent force range determined by uniformly distributed load in comparison with simulations.

#### 4.5. Continuous traffic

As described in the previous section, the randomness of vehicles can be simplified having a uniformly distributed load with a defined length; however, a vehicle-by-vehicle traffic condition is considered so far, and the results of former traffic simulations [8,9] have been confirmed that a vehicle-by-vehicle traffic model is not precise enough. The main difference with continuous traffic is that the probability of having several trucks on the bridge exists.

In order to quantify this probability, the distribution of the intervals between heavy vehicles (GVW over 100 kN) is calculated for the WIM stations of Switzerland in 2009 (see Fig. 13). The accuracy of time measurements for each passage in the available databases is one second. Therefore, the minimum possible measured distance between vehicles (head to head) is about 25 m assuming the heavy vehicles are circulating with speed of 90 km/h. Also, the frequency of the time intervals are shown up to 20 s, meaning about 500 m between two following heavy vehicle, which is generally not accounted for in the simultaneity effect. The values given in the parentheses are the Annual Average Daily Heavy Vehicle Traffic (AADHVT) corresponding to the WIM station. Fig. 13 confirms that the number of heavy vehicles with small intervals increases with heavy vehicle traffic volume, AADHVT. In Fig. 13, the shape of the distribution functions are similar for different WIM stations except for Götthard station. It can be pointed out that the Götthard station is the only station with one traffic lane in each direction. In this case, the minimum distance between heavy vehicles must be 100 m based on the Swiss traffic regulations, which makes heavy vehicles spacing longer.

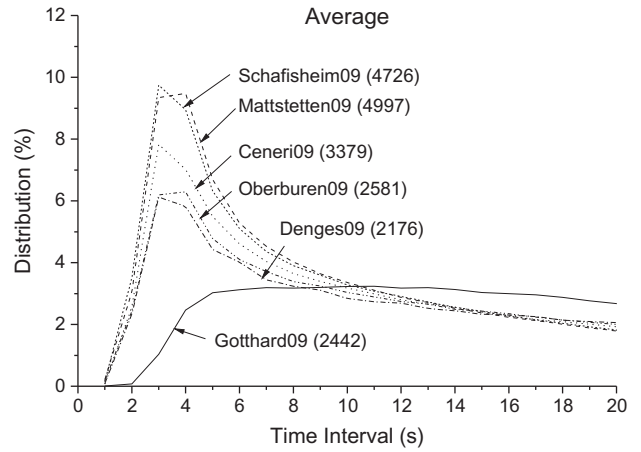


Fig. 13. Frequency of time intervals between heavy vehicles for different WIM stations of Switzerland in 2009.

The equivalent force range for continuous traffics condition can be determined based on the frequency of distances between heavy vehicles which is a function of traffic volume. However, such an approach leads to more complications in damage equivalence factors, in spite of providing more precise estimation of damage sum, which contradicts the concept of damage equivalence factors. Therefore, a simplified approach must be considered that can include the effect of continuous traffic within the damage equivalence factors. Such an approach is given in the following section.

#### 4.6. Proposition of parameters

The shortcomings of damage equivalence factors, as mentioned in the previous sections, limit the possibilities of modifications to fatigue equivalent length and fatigue load model. The damage equivalence factor for the distributed load with limited length can be obtained by dividing both sides of Eq. (10) by force range due to the passage of fatigue load model on bridge:

$$\lambda_{Dist} = \frac{\Delta F_{R,Dist}}{\Delta F_{FLM}} \times Q_m \left( \frac{N_{obs} N_{inf}}{2 \times 10^6} \right)^{1/5} \quad (11)$$

where  $\Delta F_{FLM}$  is the bridge response due to a given fatigue load model. Considering the imaginary influence lines, for instance, all cycles due to passage of vehicles over the I3 case is twice of I0 case. In order to have the same damage equivalence factor for both cases, thus the equivalent forces can be simply divided by the difference between the maximum and minimum,  $\Delta_{inf}$ . Obviously, the response of bridges due to passage of a single axle load with load intensity of  $P$  bears  $\Delta F_{FLM} = \Delta_{inf} \times P$ . Whereas the main purpose of fatigue load model is only to uniform the ordinate of influence lines, a single axle fatigue load model can perform this function. Therefore, a single axle fatigue load model with the weight of 480 kN, same as total weight of fatigue load model in Eurocode [2], can be proposed. In fact, weight of fatigue load model does not alter equivalent force range, it only moves the curve of damage equivalence factor up and down.

The first part of Eq. (11) can be rewritten as follows:

$$\frac{\Delta F_{R,Dist}}{\Delta F_{FLM}} = \frac{P \int_{x_a}^{x_b} [I.L.F_R(x)] dx}{\Delta_{inf} \times P} \quad (12)$$

The integral part is summation of some area parts under influence line. However, a vehicle-by-vehicle traffic is considered in determination of the uniformly distributed load with a defined length. Reading Section 4.5 one can notice that indeed the whole area under influence line can be effective since the probability of

having several trucks over bridge exists. In addition, the division of the integral part by  $\Delta_{inf}$  has the unit of length. Since the  $\lambda_1$  factor is function of length, the parameter of fatigue equivalent length,  $L_\lambda$ , can be proposed as:

$$L_\lambda = \frac{A_{inf}}{\Delta_{inf}} \quad (13)$$

where  $A_{inf}$  is the absolute sum of area under influence line and  $\Delta_{inf}$  is the difference between the maximum and minimum values. The fatigue equivalent length for five different influence lines obtained from Eq. (13) is shown in Fig. 14. Also, the critical span length corresponding to the [3] is given in the Fig. 14 for comparison.

It must be noted that an equal importance is given to all area sections under influence line in Eq. (13); however, in reality, the importance of different area sections under influence lines depends on both frequency of the distances between trucks and influence line length and shape. As far as the frequency of distances between trucks vary traffic by traffic,  $L_\lambda$  is traffic dependent. Moreover, for short span bridge lengths as described in Section 4.2, the uniformly distributed load is not accurate enough. Nevertheless, a very simple formula is given here for  $L_\lambda$  to make it useful in the framework of damage equivalence factors. The range of validity of the proposed definition as well as the resulting damage equivalence factors must be controlled with several influence lines. This is performed in the next section.

The powered mean of gross vehicles weight,  $Q_m$ , and the number of heavy vehicles,  $N_{obs}$ , are also important for determination of damage equivalence factor. A base value equal to the base value considered in the Eurocodes [2,3] can be taken for calculation of the  $\lambda_1$  factor, then in agreement with the Eurocodes,  $\lambda_2$  can be applied for modification of traffic volume.

The number of repetition in an influence line,  $N_{inf}$  is an important parameter that can be effective in determination of damage equivalence factors. Establishing any general rule for determination of damage equivalence factors regardless of  $N_{inf}$  effect cannot be generalized. For the bridges with continuous girders, the maximum effect of repetition in the influence line is likely to corresponds to mid-support negative moment of two-span continuous bridges. In these cases, the effect of  $N_{inf}$  on damage equivalence factors is  $2^{1/5} = 1.15$ , assuming effective slope of SN curve is 5. For taking into account the repetition effect for a given influence line, a new general partial damage equivalence factor can be introduced as follows:

$$\lambda_5 = \frac{1}{\Delta_{inf}} \left( \sum_{i=1}^{N_{inf}} (\Delta_{inf,i})^m \right)^{\frac{1}{m}} \quad (14)$$

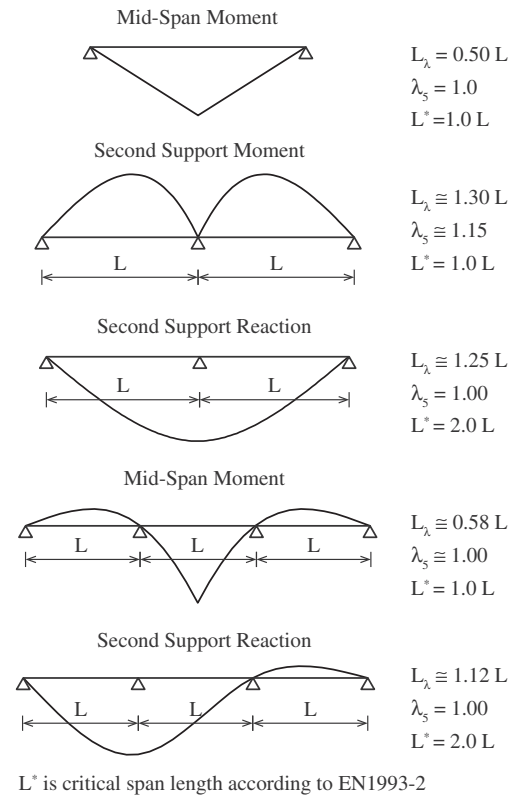


Fig. 14. Fatigue equivalent length for some sample influence lines.

where  $\Delta_{inf,i}$  is the  $i$ th cycle range of the influence line,  $\Delta_{inf}$  is the maximum cycle range (or the distance between the maximum and the minimum values) of the influence line  $m$  is the effective slope of fatigue resistance curve, which can be taken 5 for steel details under direct stress. The  $\lambda_5$  factor for five different bridge cases obtained from Eq. (14) is given in Fig. 14, though for most cases it is almost equal to 1.0.

## 5. Evaluation of proposed method

In order to evaluate the accuracy of proposed parameters, the continuous traffic simulation for several bridge cases are performed and damage equivalence factors with respect to the new proposed modifications are determined. The traffic simulation

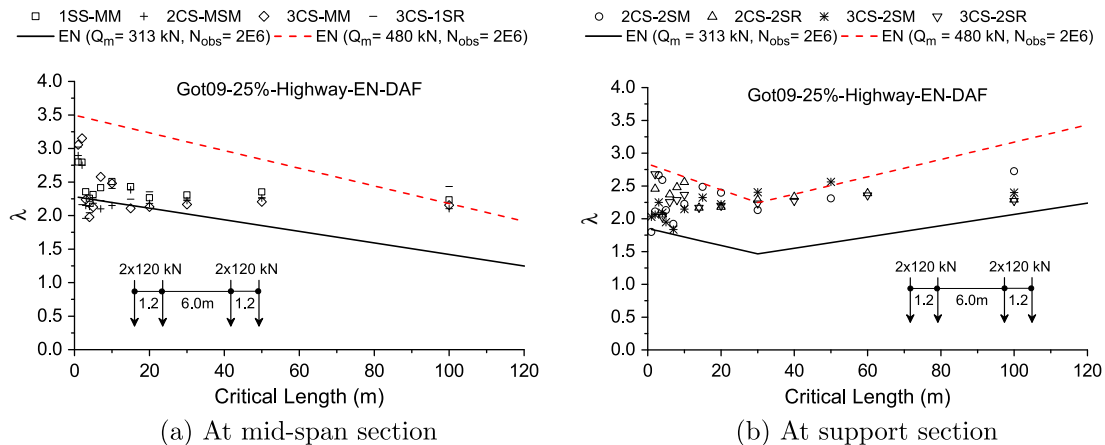


Fig. 15. Damage equivalence factor for different bridge cases in comparison with Eurocode.

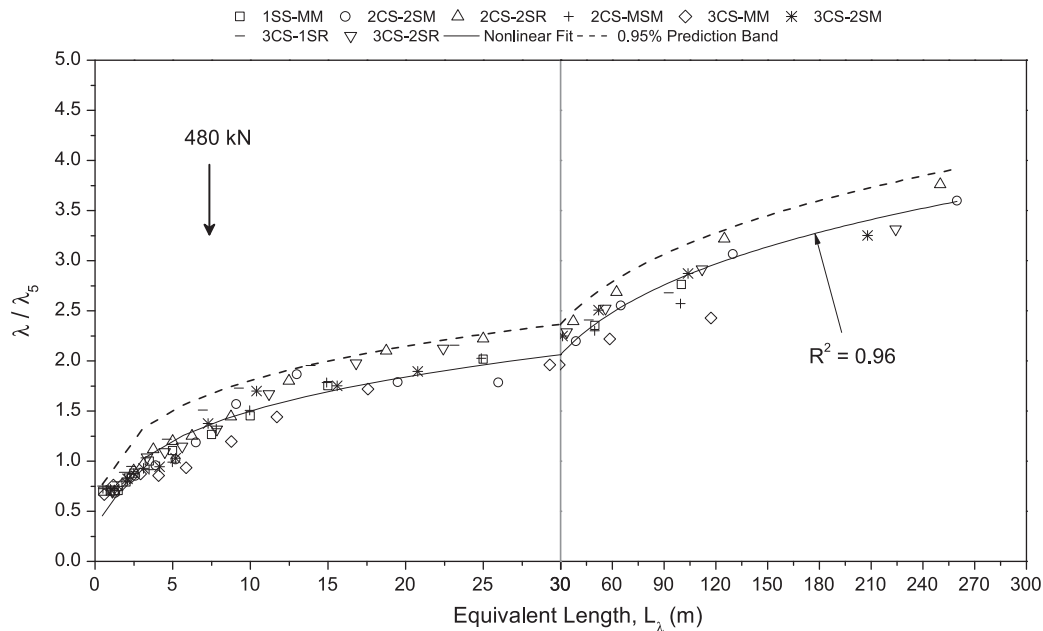


parameters assumptions as well as simulation method are explained in Section 3. Damage equivalence factors for the following static systems and detail locations are analyzed:

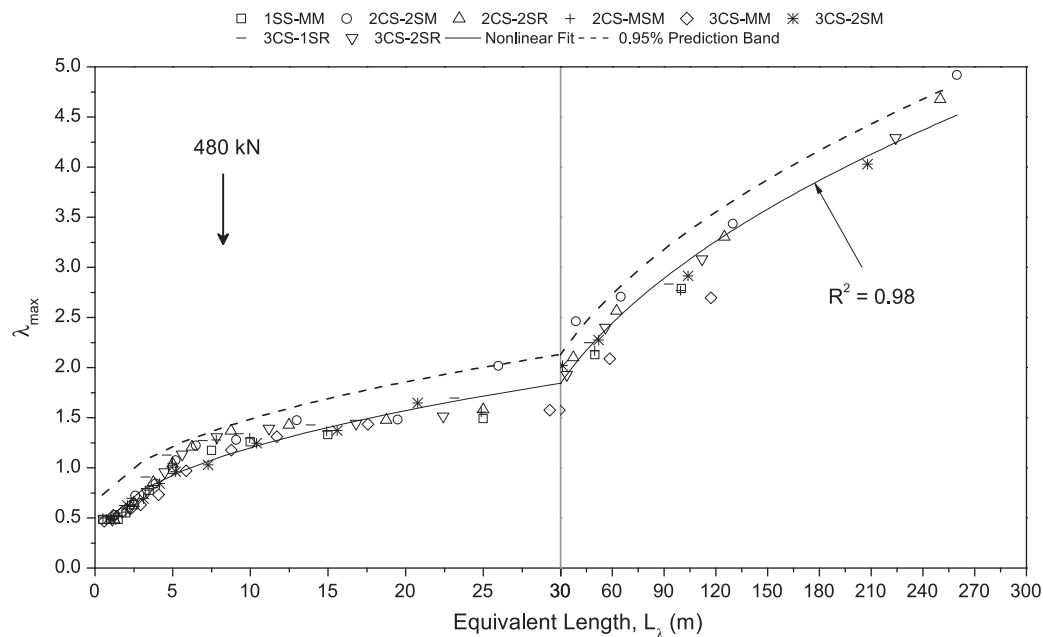
- single-span bridges, mid-span moment (1SS-MM),
- two-span continuous bridges with equal spans length, negative moment of second support (2CS-2SM) and reaction (2CS-2SR), as well as mid-span moment (2CS-MSM),
- three-span continuous bridges with equal spans length, mid-span moment of second span (3CS-MM), second support moment (3CS-2SM), as well as first and second supports reaction (3CS-1SR and 3CS-2SR).

For all static systems, the bridges span length ranges from 1 m to 200 m.

Firstly, the analysis are performed to compare damage equivalence factors resulting from simulations with the Eurocodes, as illustrated in Fig. 15; the fatigue load model (FLM3) is also plotted for clarity. The corresponding damage equivalence factors of the [3] are also shown in Fig. 15. Since the average gross weight of heavy vehicles on station Götthard (2009) is 313 kN and the number of simulated heavy vehicles is 2'000'000, partial equivalence factor,  $\lambda_2 = 313/480 \times (5/20)^{(1/5)}$ , is multiplied to calculate the  $\lambda$  of the code. Fig. 15 shows that the damage equivalence factor obtained for both mid-span and support sections are above the curve of



(a)  $\lambda$



(b)  $\lambda_{max}$

Fig. 16. Damage equivalence factors based on the proposed method for different influence lines (scale of  $L_x$  axis changes at 30 m).

the code, expressing the code might be non-conservative. The safety margin of the Eurocode curves (mid-span and support) depend on the span length and bridge static system, which is not desirable.

Afterward, the damage equivalence factor,  $\lambda/\lambda_5$ , for different influence lines based on the new hypothesis are illustrated in Fig. 16a; the average value non-linear fit of the results as well as the 95% prediction band are plotted. The single axle fatigue load model is also shown for clarity. The R-square ( $R^2$ ) is 0.96 and the dispersion of the results is clearly less than the Eurocode, despite the simple definition of the fatigue equivalent length and fatigue load model. Also, the results obtained for longer fatigue equivalent lengths (up to 280 m) are in accordance with the proposed method, which shows the robustness of the proposed methods for large lengths as well. Therefore, the proposed fatigue equivalent length is an appropriate representation for any influence line. In addition, the proposed fatigue load model does not cause dispersion of the damage equivalence factors in the short span region. It is worthy of note that a lower limit exists for damage equivalence factors in very short span lengths ( $L \leq 5$  m), which is related to the same description given in Section 4.2. Whereas the equivalent force range based on the axles load distribution at Gotthard is 249 kN and the intensity of the single axle FLM is 480 kN, this minimum value can be determined as 0.73 (as can be seen on Fig. 16a), assuming a DAF equal to 1.4.

In order to assess consistency of the proposed modifications with maximum damage equivalence factor,  $\lambda_{max}$  is determined by applying the same traffic simulations for various bridge cases. Fig. 16b shows the same modifications can be applied to  $\lambda_{max}$  as well. This justifies that the same approach could be followed to find effective parameter for determining the maximum “frequent” load response for different bridge influence lines.

## 6. Conclusions

The concept of damage equivalence factors and its main shortcomings are described. To develop a proper solution for improving the accuracy of damage equivalence factors, an analytical approach in conjunction with a traffic simulation program is considered. According to the analytical solutions, the fatigue equivalent length notion as well as a new fatigue load model are proposed. For a given influence line, the fatigue equivalent length equals the area under the influence line divided by the difference between the maximum and minimum values of the influence line. The proposed fatigue load model comprises a single axle weighting 480 kN. In order to generalize the application of the proposed fatigue equivalent length and the corresponding damage equivalence factor, a new partial damage equivalence factor ( $\lambda_5$ ) is needed and defined. The  $\lambda_5$ -factor takes into account the effect of repetition that occurs in some influence lines. The validity of the proposed modifications for the maximum damage equivalence factor is also investigated. Based on the work presented in this paper, the following conclusions are made:

- The equivalent force range for very short span lengths can be obtained from the damage summation based on the axle frequency of a given traffic and the number of repetitions for a given influence line. Similarly, for very long span lengths, the damage summation based on the gross vehicle weight of the traffic gives the equivalent force range.
- An equivalent vehicle as defined for mid-range span lengths, can properly represents the vehicle-by-vehicle traffic circulation. Loosing some accuracy, the equivalent vehicle can be simplified as a uniformly distributed load over a defined length (found to be about 9.5 m).

- Although  $\lambda_5$  is close to one in most cases, it must be considered for example in the case of two span bridge, second support moment, where the influence line repeats itself.
- The proposed parameters reduce the scatter in the damage equivalence factors for all the different influence lines studied, in spite of very simple and generic definitions for the fatigue load model and fatigue equivalent length.
- The results show that the proposed parameters also applies to  $\lambda_{max}$ .
- The damage equivalence factors based on the Eurocode might be non-conservative for some bridge cases, especially, when the  $\lambda_2$ -factor is used to adapt the traffic volume.

Further studies are of interest to find out the effect of different traffic conditions on damage equivalence factors. In the current paper, only one traffic condition is used for determining the damage equivalence factor. However, additional simulations with different traffic conditions are recommended to find out the effect of traffic parameters on damage equivalence factors. In addition, in the current paper, the fatigue resistance curves corresponded to steel details under direct stress. Further studies were carried out for determining damage equivalence factors for other SN curves, e.g. steel details under shear stress, shear studs, concrete rebars [16].

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